1up

NASA TECHNICAL TRANSLATION NASA TT F-15,546

RESTORATION OF THE SHAPE OF ANALOG SIGNALS BY DISCRETE READINGS

V.P. Yevdokimov and L.I. Kolesnikov

(NASA-TT-F-15546) RESTORATION OF THE SHAPE OF ANALOG SIGNALS BY DISCRETE READINGS (Kanner (Leo) Associates) 16 p HC $4.00 CSCL 17B

N74-21797

Unclas

G3/07 36845

Translation of "Vosstanovleniye formy analogovykh signalov po diskretnym otschetam," Academy of Sciences of the USSR, Institute of Space Research, Moscow, Report Pr-135, 1973, 19 pp.

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON, D.C. 20546 APRIL 1974

STANDARD TITLE PAGE

1. Report No. NASA TT F-15,546	2. Government Accession No.	3. Recipient's Catalog No.
4. Title and Subtitle RESTORATION OF THE SHAPE OF ANALOG SIGNALS BY DISCRETE READINGS		5. Report Date April 1974
		6. Performing Organization Code
7. Author(s) V.P. Yevdokimov and L.I. Kolesnikov		8. Performing Organization Report No.
		10. Work Unit No.
9. Performing Organization Name and Address Leo Kanner Associates, P.O. Box 5187, Redwood City, California 94063		11. Contract or Grant No. NASw-2481
		13. Type of Report and Period Covered Translation
12. Sponsoring Agency Name and Address NATIONAL AERONAUTICS AND SPACE ADMINISTRATION, WASHINGTON, D.C. 20546		14. Sponsoring Agency Code

15. Supplementary Notes

Translation of "Vosstanovleniye formy analogovykh signalov po diskretnym otschetam," Academy of Sciences of the USSR, Institute of Space Research, Moscow, Report Pr-135, 1973, 19 pp.

16. Abstract The model of a signal employing a set of possible correlation (spectral) functions is introduced. The restoration of the shape of a continuous signal by discrete readings using line filtration is examined. The precision criterion is the root-mean-square error of the restoration of the shape of the signal between readings. Line filters with finite storage (restoring functions) of increasing complexity are examined, as well as an optimal, physically unrealizable line filter. The restoration errors are calculated for different interrogation frequencies (the number of points in the correlation interval) and different restoring functions. Tables and graphs are presented for comparing errors, evaluating the utility of increasing filter complexity, and selecting filter type.

17. Key Words (Selected by Author(s))	18. Distribution Statement Unclassified - Unlimited

19. Security Classif. (of this report) Unclassified	20. Security Classif. (of this page) Unclassified	21. No. of Pages 16	22. Price 4.00

NASA-HQ

RESTORATION OF THE SHAPE OF ANALOG SIGNALS BY DISCRETE READINGS

V.P. Yevdokimov and L.I. Kolesnikov

1. Formulation of the Problem /3*

In the transmission of messages by systems with time separation of channels, continuous signals are subjected to time discretization. The resulting discrete regular sequence of readings makes it possible to restore the shape of the transmitted signal at the point of reception with a certain error that depends on the frequency of interrogation, the shape and width of the signal spectrum, and the method of restoration. In the following analysis, it is assumed that the transducer and distortion noise in the transmission channel is low and can be neglected.

Let us write the restoration error at each instant of time:

$$E(t) = x(t) - x_r(t)$$

$x(t)$ is the true signal, and
$x_r(t)$ is the restored signal.

The criteria for estimating the difference in the shape of the true and the restored signals can vary:

1) The root mean square error of the restoration of the shape at any point between the discrete readings; and

2) The probability that for a certain point between readings the restoration error does not exceed a specified value.

In this study, the following main problems are solved: /4

*Numbers in the margin indicate pagination in the foreign text.

1) the error of restoring the shape of signals is calculated using different correlation functions, with variation in the interrogation frequency and in the weighting function of the restoring filter; and

2) the results of computations are compared to arrive at recommendations for selecting the method of restoring signals, that is, an evaluation of the utility of greater complexity in the restoring filter in order to reduce errors.

The calculation in the investigation was made on the basis of the first criterion.

2. Signal Models and Methods of Restoring Shape

Let us examine signals that are steady random processes with specified correlation (spectral) functions. It is useful to select for these signals a single parameter providing for the possibility of comparing the results when the interrogation frequency is varied. We will use as this parameter the interval of correlation of the process, which we will define as $T_K = (1/2)F_{ef}$, where F_{ef} is the effective width of the signal spectrum. Let us define the generalized interrogation frequency as equal to the ratio of the interrogation frequency to the doubled effective width of the spectrum and indicating the number of interrogation points in the correlation interval of the process:

$$f = \frac{F_0}{2F_{ef}} = \frac{T_K}{T_0}$$

Let us consider as signal models the following processes, exhibiting the following correlation functions and power spectra:

1) White noise passed through a single RC filter: /5

$$R(\tau) = \sigma^2 e^{-\alpha|\tau|}, \quad S(\omega) = \frac{2\alpha}{\alpha^2+\omega^2}, \quad \alpha = 4F_{ef};$$

2) White noise passed through two series-connected identical RC filters:

$$R(\tau)=\sigma^2(1+\alpha|\tau|)e^{-\alpha|\tau|},\quad S(\omega)=\frac{4\alpha^3}{(\alpha^2+\omega^2)},\quad \alpha=8F_{ef}$$

3) White noise passed through three series-connected RC filters:

$$R(\tau)=\sigma^2\left[1+\alpha|\tau|+\frac{(\alpha\tau)^2}{3}\right]e^{-\alpha|\tau|},\quad \overset{0}{S}(\omega)=\frac{16\alpha^5}{3(\alpha^2+\omega^2)^3},\quad \alpha=\frac{32}{3}F_{ef}$$

4) White noise passed through a filter with a gaussian frequency characteristic (the limiting characteristic in the series connection of a large number of RC filters):

$$R(\tau)=\sigma^2 e^{-\alpha\tau^2},\quad S(\omega)=\sqrt{\frac{\pi}{\alpha}}\,e^{-\frac{\omega^2}{4\alpha}},\quad \alpha=4\pi F_{ef}^2;$$

5) White noise passed through a filter with a square-wave frequency characteristic with a cut-off frequency $F = F_{ef}$:

$$R(\tau)=\sigma^2\frac{\sin 2\pi F\tau}{2\pi F\tau}.$$

We will assume that the restoration of the signal shape based on discrete readings is made with an interpolating line filter with a finite storage, whose weighting function is W(t):

$$x_r(\varepsilon)=\sum_{m=-N+1}^{N} x(mT_0)\,W(mT_0,\varepsilon),$$

where m is the number of interpolation nodes, /6
M = 2N is the total number of interpolation nodes,
T_0 is the time between interrogations, and
ε is the fraction of the time interval between interrogations, $0 \leq \varepsilon \leq 1$.

Let us examine the following methods of restoring the signal between adjoining readings:

1) Stepwise restoration:

   a) without shift -- the horizontal line of the estimate is drawn to the right from each sample for the time T_0: and

   b) with a shift of $T_0/2$ -- the horizontal line of the estimate is drawn to the right and left of each sample for the time $T_0/2$.

2) Piecewise-linear restoration by joining neighboring readings with straight lines.

3) Restoration by a finite set of functions of the form $\frac{\sin x}{x}$.

4) Optimal linear restoration by a physically unrealizable filter, using for the restoration an infinite number of samplings on both sides of the interpolation interval.

3. Criterion of the Root-Mean-Square Error of Restoration

The root-mean-square error of shape restoration for each time instant between readings can be calculated by the following formula [1]:

$$\overline{E^2(\varepsilon)} = R(0) - 2\sum_{-N+1}^{N} R[(m-\varepsilon)T_0]W[(m-\varepsilon)T_0] + \sum_{m=-N+1}^{N}\sum_{l=-N+1}^{N} R[(m-l)T_0]W[(m-\varepsilon)T_0]W[(l-\varepsilon)T_0],$$

where m and l are the numbers of the interpolation nodes involved /7
in restoration.

We will calculate the error in percentages of the scale of signal change $\pm 3\sigma (6\sigma)$:

$$E(\varepsilon) = \frac{\sqrt{\overline{E^2(\varepsilon)}}}{6}.$$

The interpolation error takes on a maximum in the middle of the interval between reading for all kinds of interpolation, except for stepwise interpolation without shift, that is, when $\varepsilon = 0.5$ (for stepwise interpolation without shift, for $\varepsilon = 1$).

The weighting functions of the restoring filters for these restoration methods are of the following form:

1) Stepwise restoration ($M = 1$, $m = 0$):

$$W(\varepsilon) = 0$$

2) Piecewise-linear restoration ($M = 2$, $m = 0, 1$):

$$W[(m-\varepsilon)T_0] = \begin{cases} \varepsilon, & m=0 \\ 1-\varepsilon, & m=1 \end{cases}$$

3) Restoration by the functions $\frac{\sin x}{x}$ (the number of nodes M is a variable; restoration is carried out with a filter that is matched with the interrogation frequency $F_0 = 2\,F_{ef}$):

$$W[(m-\varepsilon)T_0] = \frac{\sin \pi(m-\varepsilon)}{\pi(m-\varepsilon)}.$$

The determination of the error for the optimal line interpolation (averaged over all ε within the interval between readings) was made using the following formula, obtained in [2]):

$$\overline{E^2} = \frac{1}{\pi}\int_0^\infty S(\omega)\left[1 - \frac{S(\omega)}{T_0\,\varphi(\omega)}\right]d\omega,$$ /8

where $S(\omega)$ is the energy spectrum of the restored process, and

$\varphi(\omega) = \frac{1}{T_0}\sum S(\omega - i\omega_0)$ is the energy spectrum of the discrete random process, consisting of readings following at a frequency ω_0.

4. Criterion of the Probability of Maximum Restoration Error

Use of this criterion is facilitated for normal random processes.

Since line filters are used for the restoration of the signal shape, the restoration error at each time instant ε is a random variable with normal distribution of probabilities and with a dispersion defined by the formula presented for $\overline{E^2(\varepsilon)}$.

Accordingly, the probability that at each point of the interval between the readings the error does not exceed a specified value can be easily calculated by using the tables of the normal distribution of probabilities.

5. Results of Calculations and Conclusions

The results of calculating the maximum root-mean-square normalized error of the restoration of signals using different correlation functions are given in the tables. /9

Table 1 contains the errors of stepwise restoration. The errors prove to be significant in magnitude for any signal model even at high interrogation frequencies. Therefore the use of this restoration method appears disadvantageous.

Table 2 and Fig. 1 present the errors for piecewise-linear restoration. The errors prove to be much smaller, and the advantage with respect to stepwise restoration increases with increase in the interrogation frequency.

When the signal spectrum is changed from model 1 to model 5, an increasing suppression of the high-frequency "tails" in the spectrum occurs. Thus, increasing complexity in the filter shaping the signal spectrum leads to a reduction in the errors for any interrogation frequencies.

Restoration using the functions $\frac{\sin x}{x}$ makes it possible, without changing the weighting function of the filter, to involve a different number of nodes in the interpolation process. The results of calculating the errors when the number of nodes was varied from 20 to 200 are given in Table 3. The dependence of the errors on N is shown in Fig. 2. It is interesting to note that for the first three signal models, at the interrogation frequencies considered no appreciable reduction in the errors was observed with an increase in the number of nodes participating in the restoration. Since as $N \to \infty$ there is a limiting nonzero restoration error, it can be concluded that there is a fairly rapid convergence of the series with the functions $\frac{\sin x}{x}$. This implies that for restoration with the functions $\frac{\sin x}{x}$ in practice it is not advantageous to use more than 10 - 20 interpolation nodes. If we consider the change in the errors with increase in the interrogation frequency and with constant node number as in Fig. 3, we can note that regardless of the signal model an increase in /10 the interrogation frequency beyond some value leads to a constant error identical in magnitude.

A comparison of piecewise-linear restoration and the restoration using the functions $\frac{\sin x}{x}$ can be made from Figs. 4 and 5. The errors with optimal linear, piecewise-linear restoration, and restoration using the functions $\frac{\sin x}{x}$, averaged over all ε values in the interval between readings, are given in Fig. 6. The comparison shows that all the way up to the shape of the spectrum determined by the forming 3 x RC filter, it is not advantageous to increase the complexity of the restoring filters, since the use of piecewise-linear restoration provides virtually the same error values as optimal linear restoration. Only for the gaussian shape of the signal spectrum are the errors of optimal linear restoration smaller compared with those under piecewise-linear restoration, where the difference increases with greater interrogation frequencies and for smaller errors. When there are increased requirements and higher precision, this difference in the error can be regarded as significant. In this case, restoration with the functions $\frac{\sin x}{x}$ significantly reduces this error down to some interrogation frequency, which depends on the number of interpolation nodes used.

Interestingly, for some unknown signal model, but belonging to the class under study, restoration with the functions $\frac{\sin x}{x}$ leads to errors that are practically the same as the errors for optimal linear restoration.

This also suggests the conclusion that there is no practical advantage in the analysis and use of the optimal linear restoration with a finite number of interpolation nodes.

In practice, the small difference between the errors yielded /11 by piecewise-linear restoration compared with more complex methods of restoration leads to the conclusion that the predominant use of piecewise-linear restoration of the signal shape between readings is advantageous.

TABLE 1. STEPWISE RESTORATION.

Kind of Restoration \ f		1	5	10	20	50	100	Kind of Correlation Function
Without	shift	22	13.5	10	7.25	4.7	3.33	RC filter
With	shift	19.8	10	7.25	5.16	3.33	2.33	
Without	shift	22.5	10.3	5.8	3.1	1.3	0.7	2×RC filter
With	shift	18.2	5.8	3.1	1.6	0.7	0.33	
Without	shift	22.7	9.4	5	2.6	1	0.5	3×RC filter
With	shift	18	5	2.6	1.3	0.5	0.26	
Without	shift	23	8	4.1	2.1	0.84	0.42	Gaussian filter
With	shift	17.4	4.1	2.1	1	0.42	0.21	
Without	shift	23.6	6	3	1.6	0.6	0.3	Line filter
With	shift	14.2	3	1.6	0.8	0.3	0.16	

TABLE 2. LINEAR RESTORATION.

Form of Correlation Function \ f	1	2	3	5	10	20	50	100
RC-filter	15.2	11.8	9.8	7.4	5.5	3.7	2.3	1,67
2×RC filter	14.3	8.02	5.17	2.77	1.09	0.4	0.1	0.04
3×RC filter	13.94	6.82	3.88	1.72	0.5	0.14	0.023	0.006
Gaussian filter	13.01	4.85	2.35	0.88	0.23	0.057	0.009	0.0023
Line filter	7.94	2.22	1.0	0.37	0.09	0.023	0.0036	0.0008

TABLE 3. RESTORATION USING THE FUNCTIONS $\frac{\sin x}{x}$.

No. \ f	1	2	3	5	10	20	Form of Correlation Function
10	17.4	12.9	10.6	8.3	5.9	4.2	RC filter
20	17.43	12.9	10.51	8.31	5.91	4.21	
50	17.45	12.9	10.64	8.33	5.92	4.22	
100	17.5	12.96	10.65	8.35	5.93	4.23	
10	16.2	8.7	5.27	2.48	1.08	0.5	2 x RC filter
20	16.3	8.78	5.32	2,42	0,98	0,39	
50	16.35	8.8	5,33	2.42	0.96	0.35	
100	16.36	8.8	5,33	2.42	0.96	0.34	
5	15,87	6,95	3.4	1.36	0.87	–	3 x RC filter
10	16,02	7.03	3.4	1.19	0.42	–	
20	16.1	7.08	3.42	1.16	0.28	–	
10	15.1	3.06	0.59	0.39	0.38	0.37	Gaussian filter
20	15.15	3.5	0.47	0.19	0.189	0.186	
50	15.2	3,6	0,45	0.08	0.07	0.04	
100	15.25	3.67	0.43	0.039	0.038	0.037	
10	2,37	0.42	0.4	0.38	0.37	0.37	Line filter
20	1.67	0,21	0.2	0.19	0,187	0,186	
50	1.05	0,083	0,08	0,08	0.08	0,08	
100	0.78	0,042	0,041	0.038	0.037	0.037	

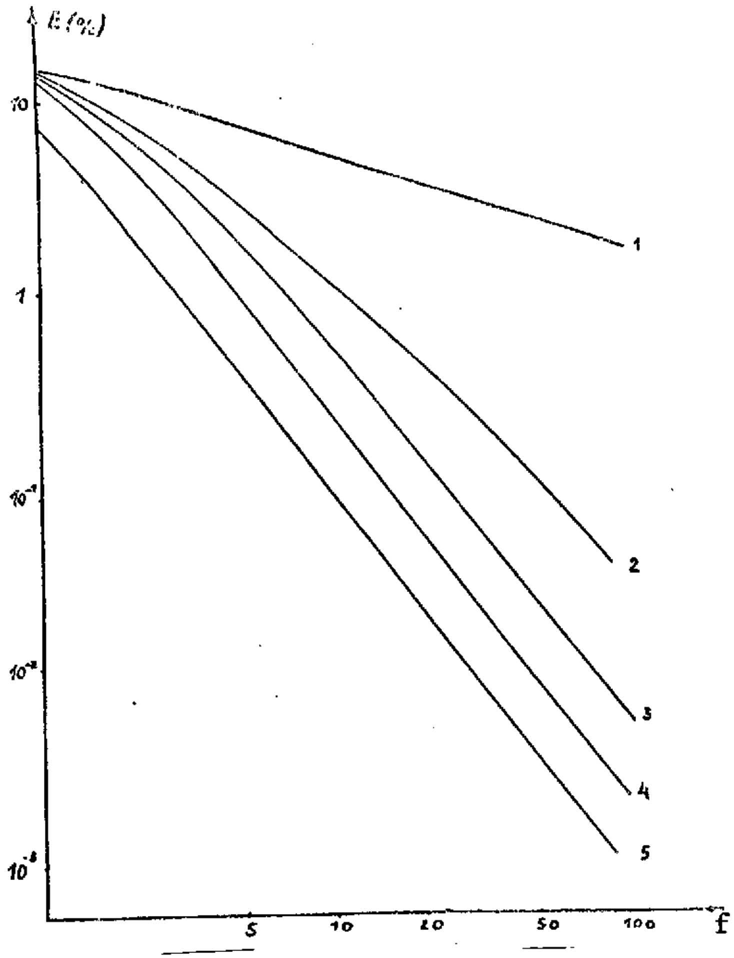

Fig. 1. Errors in piecewise-linear restoration (PL). 1. RC filter; 2. 2 x RC filter 3. 3 x RC filter; 4. gaussian filter; 5. linear filter.

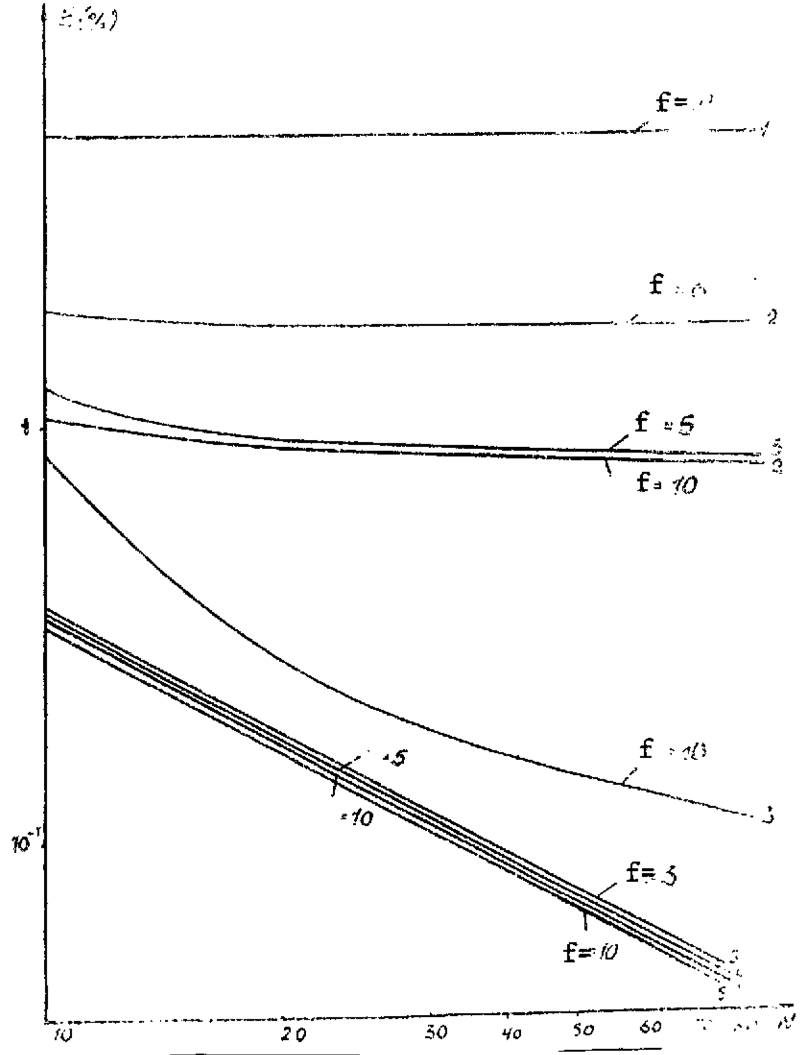

Fig. 2. Errors in restoration using the functions $\frac{\sin x}{x}$ with change in the number of nodes ($M \cong 2N$).

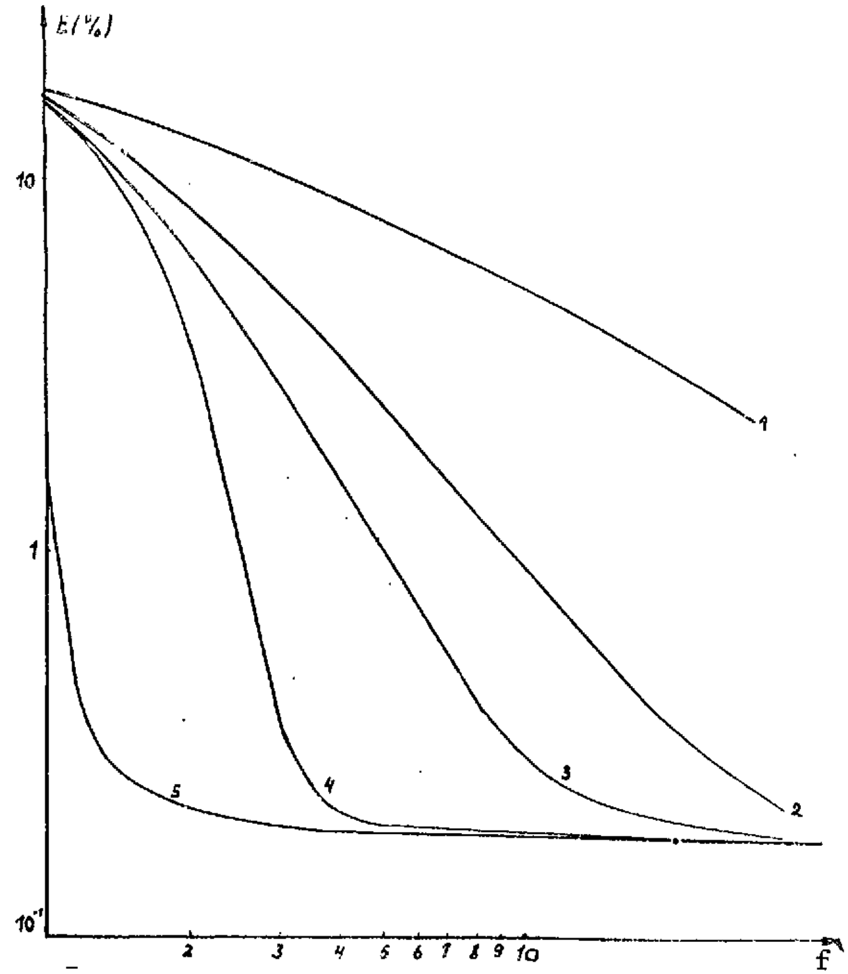

Fig. 3. Errors in restoration using the functions $\frac{\sin x}{x}$, N = 20. 1. RC filter; 2. 2 x RC filter; 3. 3 x RC filter; 4. gaussian filter; 5. square-wave filter.

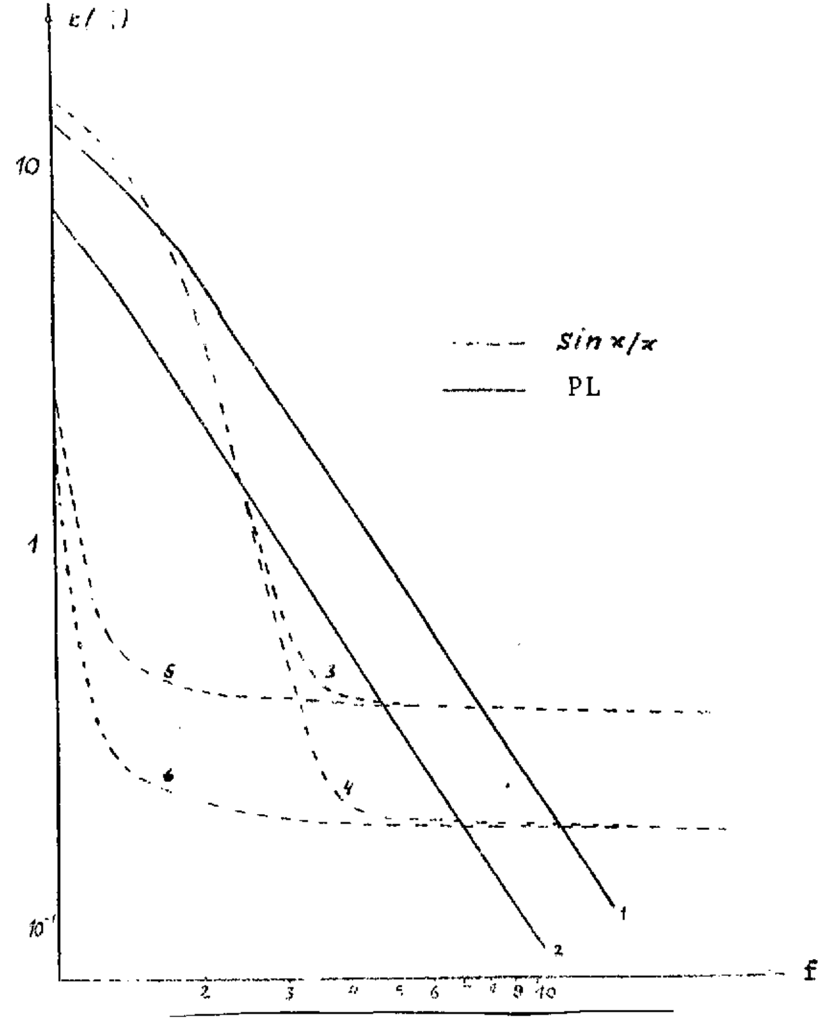

Fig. 4. Comparison of restoration methods. 1, 3, 4. Gaussian filter; 2, 5, 6. square-wave filter; 3, 5. N = 10, N - 4. 6. N = 20.

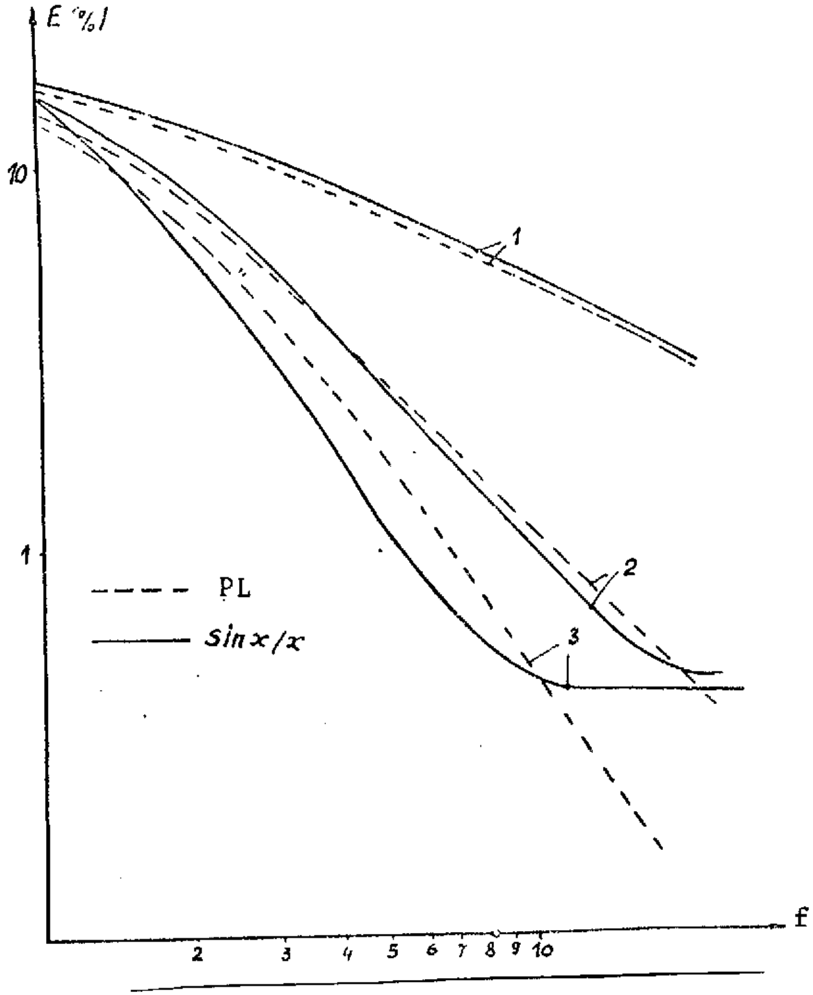

Fig. 5. Comparison of restoration methods.
1. RC filter; 2. 2 x RC filter; 3. 3 x RC filter, N = 10.

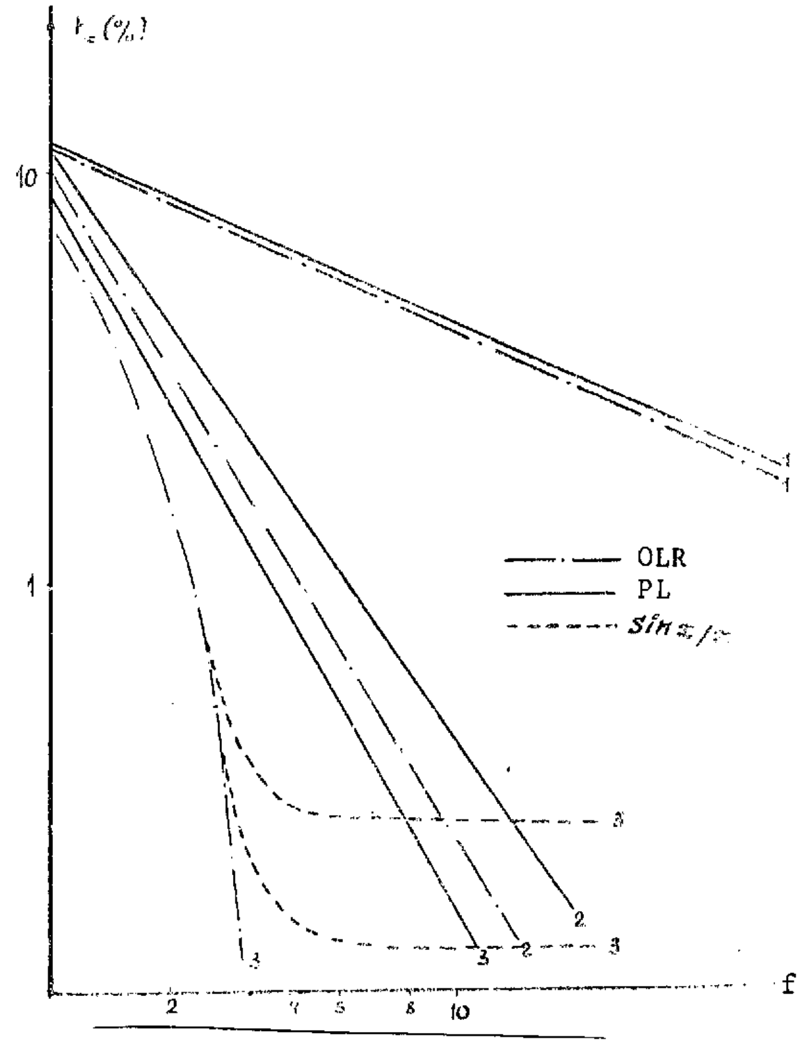

Fig. 6. Comparison with optimal linear restoration (OLR).
1. RC filter; 2. 3 x RC filter; 3. gaussian filter.

REFERENCES

1. Andreyev, N.I., Korrelyatsionnaya teoriya statisticheski optimal'nykh sistem [Correlation theory of statistically optimal systems], Moscow, "Nauka," 1966.

2. Bykov, V.V., Tsifrovoye modelirovaniye v statisticheskoy radiotekhnike [Digital simulation in statistical radio engineering], Moscow, "Sovetskoye Radio," 1971.